Tentamen Applied Signal Processing

31-01-2011

1 a $\quad x[n] = \left(\frac{1}{3}\right)^n \mu[n]$ 9

$$X[z] = \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n \mu[n]\, z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^n$$

$$= \frac{1}{1-(3z)^{-1}} \quad \text{for the ROC } \frac{1}{3z} < 1,\ 3z > 1,\ |z| > \frac{1}{3}$$

$$= \frac{3z}{3z-1}$$

zero if $3z = 0 \rightarrow z = 0$

pole if $3z - 1 = 0 \rightarrow z = \frac{1}{3}$

b $\quad x[n] = \left(\frac{1}{3}\right)^n (\mu[n] - \mu[n-10])$

$$X[z] = \sum_{n=-\infty}^{\infty}\left[\left(\frac{1}{3}\right)^n \mu[n]\, z^{-n}\right] - \sum_{n=-\infty}^{\infty}\left[\left(\frac{1}{3}\right)^n \mu[n-10]\, z^{-n}\right]$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{3z}\right)^n - \sum_{n=10}^{\infty}\left(\frac{1}{3z}\right)^n$$

$$= \frac{1}{1-\frac{1}{3z}} - \sum_{n=0}^{\infty}\left(\frac{1}{3z}\right)^n \quad \left(\frac{1}{3z}\right)^{10}$$

$$= \frac{1}{1-\frac{1}{3z}} - \frac{\left(\frac{1}{3z}\right)^{10}}{1-\frac{1}{3z}}$$

$$= \frac{3z}{3z-1} - \frac{\left(\frac{1}{3z}\right)^9}{(3z-1)}$$

$$= \frac{3z - \left(\frac{1}{3z}\right)^9}{3z-1}$$

with ROC $\frac{1}{3z} < 1$, $3z > 1$, $|z| > \frac{1}{3}$

zero if $\quad 3z - \left(\frac{1}{3z}\right)^9 = 0$

$\quad 3z = \left(\frac{1}{3z}\right)^9$

$\quad z = 0$

pole if $\quad (3z-1) = 0$

$\quad 3z = 1$

$\quad z = 1/3$

c $\quad X(z) = \dfrac{1}{1+0{,}5z^{-1}} \qquad |z| < 0{,}5$

$$= \sum_{n=0}^{\infty} \left(\frac{-1}{2z}\right)^n$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{-1}{2z}\right)^n \mu(n) = \sum_{n=-\infty}^{\infty} \left(\frac{-1}{2}\right)^n \mu(n) \; z^{-n}$$

$$\Rightarrow x(z) = \left(\frac{-1}{z}\right)^n \mu(n)$$

d $\quad X(z) = \dfrac{1}{1+0{,}5z^{-1}}$

$$= \sum_{n=0}^{\infty} \left(-\frac{1}{2z}\right)^n$$

$$= \sum_{n=-\infty}^{\infty} \left(-\frac{1}{2z}\right)^n \mu(n)$$

$$\Rightarrow x(z) = \left(-\frac{1}{2}\right)^n \mu(n)$$

c $\quad X(z) = \dfrac{1}{1+0{,}5z^{-1}} = \dfrac{1}{1+\frac{1}{2z}} = \dfrac{2z}{2z+1} = \dfrac{1}{1+2z}\, 2z$

$$x[z] = \sum_{n=0}^{\infty} (-2z)^n \cdot 2z$$

$$= -\sum_{n=0}^{\infty} (-2z)^n \cdot -2z$$

$$= -\sum_{n=1}^{\infty} (-2z)^n$$

$$= -\sum_{n=-\infty}^{-1} \left(-\frac{1}{2z}\right)^n$$

$$= -\sum_{n=-\infty}^{\infty} \left(-\frac{1}{2}\right)^n \mu(-n-1) \; z^{-n}$$

$$\Rightarrow x[n] = -\left(-\frac{1}{2}\right)^n \mu(-n-1)$$

2 $x_a(t) = A\cos(\Omega_0 t + \phi)$

$x[n] = x_a(nT) = \cos(\Omega_0 nT) \leftarrow$ periodic for every $T \neq 0$?

Nyquist:

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$f_{sample} \geq 2 f_{max}$

$T = \frac{1}{f}$ $\qquad\qquad T = \frac{2\pi k}{n\Omega_0}$

$\frac{1}{T} \geq 2\Omega_0$

$T \leq \frac{1}{2\Omega_0}$.

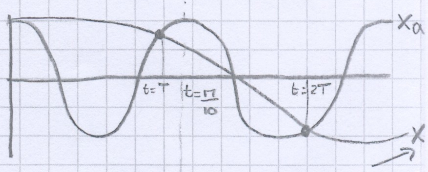
(if $\Omega_0 = 20$ radians, I can not take a $\cos(\Omega_0 nT)$.)

$x[n] = \cos(\Omega_0 nT) = \cos(\frac{5}{2}\pi\ n)$

$\rightarrow$ fundamental period:

$\cos(20 \cdot \frac{\pi}{8}) - \cos(\frac{2\pi}{P_f} \cdot \frac{\pi}{8}) = 0$

$\rightarrow P_f = \cancel{40}\ \boxed{\frac{\pi}{2k}}$



cosine with fundamental ~~frequen~~period.

3 a $a = X[z] + b + c$

$b = a \cdot -0{,}4 \cdot z^{-1}$

$c = a \cdot 0{,}12\, z^{-2}$

$a = X[z] - 0{,}4 z^{-1} a + 0{,}12\, z^{-2} a$

$a + 0{,}4 z^{-1} a - 0{,}12 z^{-2} a = X[z]$

$a = \frac{X[z]}{1 + 0{,}4 z^{-1} - 0{,}12 z^{-2}}$

$y[z] = a - 2z^{-1} a$ $\qquad$ 9

$= \frac{X[z]}{1 + 0{,}4 z^{-1} - 0{,}12 z^{-2}} - \frac{2z^{-1} X[z]}{1 + 0{,}4 z^{-1} - 0{,}12 z^{-2}}$

$H[z] = \frac{1 - 2z^{-1}}{1 + 0{,}4 z^{-1} - 0{,}12 z^{-2}}$ $\qquad$ 8

3b

$$\frac{1-2z^{-1}}{1+0,4z^{-1}-0,12z^{-2}} = \frac{z^2-2z}{z^2+0,4z-0,12}$$

zero if: $z^2 - 2z = 0$

$z(z-2) = 0$

$z = 0$ of $z = 2$ $\cdot z$

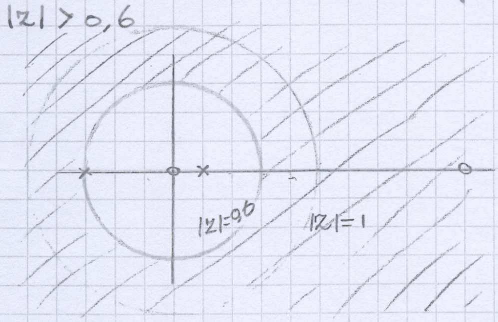
pole if: $z^2 + 0,4z - 0,12 = 0$

$D = 0,4^2 + 4 \cdot 0,12 = 0,64 = 0,8^2$

$$z = \frac{-0,4 \pm 0,8}{2}$$

$z = -0,6$ of $z = 0,2$ z

for causal systems the ROC is outside of the outermost pole, so $|z| > 0,6$



c

$$H(e^{i\omega}) = \frac{e^{2i\omega} - 2e^{i\omega}}{e^{2i\omega} + 0,4e^{i\omega} - 0,12}$$

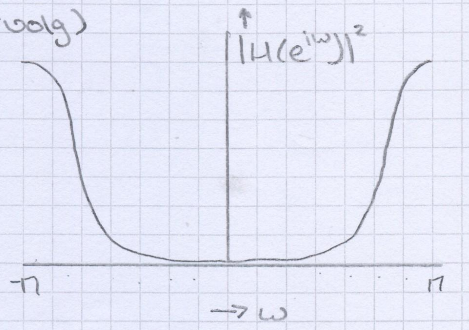
$$|H(e^{i\omega})|^2 = \frac{e^{2i\omega} - 2e^{i\omega}}{e^{2i\omega} + 0,4e^{i\omega} - 0,12} \cdot \frac{e^{-2i\omega} - 2e^{-i\omega}}{e^{-2i\omega} + 0,4e^{-i\omega} - 0,12}$$

$$= \frac{\cancel{1}2e^{i\omega} - 2e^{-i\omega} + 2}{1 + 0,4e^{i\omega} \cancel{- 0,12e^{2i\omega}} \cancel{+ 0,4e^{-i\omega}} + 0,4^2 - 0,12 \cdot 0,4e^{i\omega} \cancel{- 0,12e^{-2i\omega}} + 0,4 \cdot 0,12e^{-i\omega} \cancel{+ 0,12^2}}$$

$$= \frac{3 - 4\cos\omega}{1 + 0,4^2 + 0,12^2 + 0,8\cos\omega - 0,24\cos 2\omega - 0,096\cos\omega}$$

see also other sheet

3c (vervolg)



This is a bandstop filter.

4

$x[n] = \cos\left(\frac{\pi n}{2}\right)$, $h[n] = 2^n$

10

$n = 0, 1, 2, 3$

a DFT matrix:

$$\begin{bmatrix} \omega^0 & \omega^0 & \omega^0 & \omega^0 \\ \omega^0 & \omega^1 & \omega^2 & \omega^3 \\ \omega^0 & \omega^2 & \omega^4 & \omega^6 \\ \omega^0 & \omega^3 & \omega^6 & \omega^9 \end{bmatrix} \quad \text{with} \quad \omega = \exp\left(-\frac{2\pi i}{4}\right) = -i$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix} \begin{bmatrix} \cos(0) \\ \cos(\frac{\pi}{2}) \\ \cos(\pi) \\ \cos(\frac{3}{2}\pi) \end{bmatrix} = \begin{bmatrix} 1-1 \\ 1+1 \\ 1-1 \\ 1+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \end{bmatrix} = X(k)$$

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b $$H(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix} \begin{bmatrix} 2^0 \\ 2^1 \\ 2^2 \\ 2^3 \end{bmatrix} = \begin{bmatrix} 1+2+4+8 \\ 1-2i-4+8i \\ 1-2+4-8 \\ 1+2i-4-8i \end{bmatrix} = \begin{bmatrix} 15 \\ -3+6i \\ -5 \\ -3-6i \end{bmatrix}$$

c $$y[n] = x[n] * h[n] = \left[\frac{1\ 0\ {-1}\ 0}{1\ 2\ 4\ 8}, \frac{0\ 1\ 0\ {-1}}{1\ 2\ 4\ 8}, \frac{-1\ 0\ 1\ 0}{1\ 2\ 4\ 8}, \frac{0\ {-1}\ 0\ 1}{1\ 2\ 4\ 8} \right] = -3, -6, 3, 6$$

8

4 d $X(k)\,H(k) = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 15 \\ -3+6i \\ -5 \\ -3-6i \end{bmatrix} = \begin{bmatrix} 0 \\ -6+12i \\ 0 \\ -6-12i \end{bmatrix}$

IDFT matrix:

$$\frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} 0 \\ -6+12i \\ 0 \\ -6-12i \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -6+12i-6-12i \\ -6i-12+6i-12 \\ 6-12i+6+12i \\ -6i+12-6i+12 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -6 \\ -12 \\ 6 \\ 12 \end{bmatrix} = \begin{bmatrix} -3 \\ -6 \\ 3 \\ 6 \end{bmatrix}$$

5 $H(z) = \dfrac{z(z+2)}{(z-0{,}2)(z+0{,}6)}$ $\cancel{= \dfrac{z^2}{(z-0{,}2)(z+0{,}6)} + \dfrac{2z}{(z-0{,}2)(z+0{,}6)}}$

$$= \frac{Az}{z-0{,}2} + \frac{Bz}{z+0{,}6} = \frac{(A+B)z^2 + (0{,}6A - 0{,}2B)z}{(z-0{,}2)(z+0{,}6)}$$ 10

$\rightarrow A+B = 1$
$\hookrightarrow 0{,}6A - 0{,}2B = 2$ $\Big\}$ $A = 2{,}75$, $B = -1{,}75$

$$H(z) = \frac{2{,}75z}{z-0{,}2} - \frac{1{,}75z}{z+0{,}6} = \frac{2{,}75}{1-\frac{0{,}2}{z}} - \frac{1{,}75}{1+\frac{0{,}6}{z}}$$

for $|z| > 0{,}6$

$$= 2{,}75 \sum_{n=0}^{\infty}\left[\left(\frac{0{,}2}{z}\right)^n\right] - 1{,}75 \sum_{n=0}^{\infty}\left[\left(-\frac{0{,}6}{z}\right)^n\right]$$

$$= 2{,}75 \sum_{n=-\infty}^{\infty}\left[(0{,}2)^n \mu(n)\, z^{-n}\right] - 1{,}75 \sum_{n=-\infty}^{\infty}\left[(-0{,}6)^n \mu(n)\, z^{-n}\right]$$

$\rightarrow h[n] = 2{,}75 \; 0{,}2^n \mu(n) \; - 1{,}75 \; (-0{,}6)^n \mu(n)$

6 a Since IIR systems don't have

$$\sum_{n=-\infty}^{\infty} |h[n]|^2 \leq \infty,$$

they are not BIBO stable. 9

That is ~~an~~ disadvantage.

However, IIR systems can have a linear phase difference while FIR systems don't. This causes signal components of different frequencies to be delayed with the same ammount of time, so that the output signal looks the same.

b Since the transfer functions of analog IIR filters are known, they can be programmed in digital filters.